# On the Capacity of MRT/MRC Diversity Technique in Full-Duplex Relay System with Hardware Impairments over Rayleigh Fading Environments

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***Abstract*—This paper investigates the ergodic capacity (EC) of full-duplex (FD) relay system with maximal ratio transmission (MRT) and maximal ratio combining (MRC) techniques under the effect of hardware impairments (HI). We mathematically derive the closed-form expression of EC of the HI-FD relay system under the joint impact of HI and residual self-interference (RSI) over Rayleigh fading environments. Based on the derived expression, we can quickly obtain the EC expressions of related systems such as ideal hardware (ID) FD relay systems, HI half-duplex (HD) relay system, and ID-HD relay system. The Monte-Carlo simulation validates all analytical expressions. Numerical results indicate a strong effect of both HI and RSI on the EC of the considered HI-FD relay system. Specifically, EC reaches the capacity floor in the high signal-to-noise ratio (SNR) regime. Also, the influence of HI is more substantial when the RSI is smaller and vice versa.**

***Index Terms*—Full-duplex relay, residual self-interference, decode-and-forward, maximal ratio transmission, maximal ratio combining, hardware impairments, ergodic capacity.**

## I. INTRODUCTION

Recently, full-duplex (FD) communication systems have been widely investigated due to the benefits of FD transmission mode. Ideally, the FD technique increases the capacity of wireless systems twice compared with the traditional half-duplex (HD) transmission technique [1], [2]. Besides the advantage of high capacity, the FD transmission mode can achieve many other benefits. These include increasing the spectrum usage flexibility and throughput, reducing congestion, feedback, and end-to-end delays [1], [3]. Therefore, FD technique is exploited in various kinds of wireless communication systems such as in spatial modulation [4], [5], energy harvesting (EH) [6], cognitive radio (CR) [3], non-orthogonal multiple access (NOMA) [2] and multiple-input multiple-output (MIMO) [7].

On the other hand, the use of the relay to increase the coverage and reliability of wireless communication systems has become a hot research topic in the past decade and for the future. It is because the relays are low-cost, and that can be easily deployed in comparison with base stations [8]. The relays are expected to be used in various applications of the fifth-generation (5G) systems and beyond [9]. Nowadays, instead of traditional HD transmission, the FD transmission often applied at the relays in wireless communication systems to fully exploit the advantages of both FD and relay techniques. By using FD at the relay, the FD relay systems' capacity is much better than that of HD relay systems. However, the residual self-interference (RSI) after all self-interference cancellation (SIC) causes the error floor of outage probability (OP) and symbol error rate (SER) of FD relay systems [5], [7], [10].

Besides exploiting the FD technique at the relays, MIMO transmission has been widely used in wireless communication systems because of its capability to boost the system capacity [4]. However, utilizing multiple antennas for transmitting and receiving signals, MIMO systems suffer several disadvantages such as the complexities of hardware deployment and signal processing. In this context, maximal ratio transmission (MRT) at the transmitter and maximal ratio combining (MRC) at the receiver are considered as effective methods to solve this issue [11]. These MRT/MRC techniques can reduce the complexities in hardware deployment and signal processing while still provide MIMO systems with full diversity order. Therefore, the combination of MRT/MRC and FD techniques in the relay systems is essential for future wireless networks such as 5G and beyond [12].

In the literature, the performance of FD relay systems have been analyzed and optimized for both decode-and-forward (DF) [5] and amplify-and-forward (AF) protocols [13]. Their results indicated that the RSI causes the error floor of OP and SER of the FD relay system. Additionally, optimal power allocation for FD transmission significantly improves the capacity and reduces OP and SER of FD relay systems [5], [13], [14]. Furthermore, the joint impacts of RSI and other factors such as hardware impairments (HI) and imperfect channel state information (CSI) have also been considered in various works [13], [15], [16]. The reports in these works showed that, besides the RSI, other negative factors such as HI and imperfect CSI greatly influence the OP, SER, and EC of FD relay systems. Notably, the joint effects of HI and RSI cause the error performance of FD relay systems. However, applying MRT/MRC techniques can remarkably reduce the OP and SER of FD relay systems [17].

As discussed above, the case that HI exists in the FD relay systems has been widely considered in the literature. However, most of these works focused on single-input single-output (SISO) system or only studied relay systems with multiple-antenna FD relays and single-antenna sources and destinations [13], [15]. The relay systems with multiple-antenna sources and destinations have been only investigated via simulation results, except [17]. Unfortunately, the work in [17] only derived the OP and SER expressions of the FD relay system with

MRT/MRC techniques. Derivation of the EC expression was missing. Meanwhile, EC is an essential parameter for analyzing system performance. It is because if we want the system to reach the lowest OP/SER and highest EC, it is crucially necessary to obtain the EC expression of the FD relay system with HI. This motivates us to mathematically analyze the EC of FD relay system using MRT/MRC techniques under both HI and RSI effects. In summary, the contributions of this paper are listed as follows.

– We consider an FD relay system where multiple antennas and MRT/MRC techniques are exploited at source and destination under the effects of HI at all nodes (called as HI-FD relay system). We obtain the signal-to-interference-plus-noise-and-distortion ratio (SINDR) of the HI-FD relay system over Rayleigh fading channels.

– We successfully obtain the exact closed-form expression of the EC of the HI-FD relay system with HI and RSI. From the derived expression, we can quickly get the EC expression of related systems such as the ideal hardware (ID) FD/HD relay systems and the HI-HD relay system. We also use Monte-Carlo simulations to confirm the correctness of our analysis.

– We compare the EC of the HI-FD relay system compared with that of ID-FD, HI-HD, and ID-HD relay systems. It is shown that the joint effects of both HI and RSI have a substantial impact on the FD relay system's performance, especially in the case of high HI and RSI. Furthermore, due to the capacity floor caused by HI and RSI, it is recommended to choose an optimal transmission power for reducing the energy consumption of the considered HI-FD relay system.

The remainder of this paper is structured as follows. Section II describes the system and signal models of the considered HI-FD relay system. Section III focuses on mathematically deriving the EC of this system. Numerical results and discussion are given in Section IV. Finally, conclusions of the paper are presented in Section V.

## II. System Model

The system model of the HI-FD relay system with MRT/MRC techniques is presented in Fig. 1. Source (S) transmits signals to the destination (D) via the assistance of relay (R), where S and D are, respectively, equipped with $M$ and $N$ antennas while R is equipped with one antenna for both signal transmission and reception. S and D operate in HD mode while R operates in FD mode. Since R transmits and receives signals at the same time and on the same frequency band, the self-interference (SI) at R occurs. Therefore, R must apply all SIC techniques, including passive suppression and active cancellation to suppress the SI power.

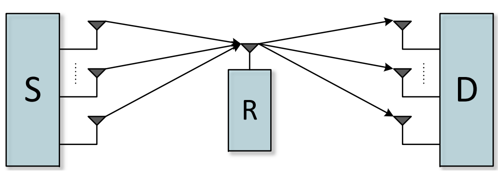


Figure 1. System model of the considered HI-FD relay system with MRT/MRC techniques.

Under the effects of HI and SI, the received signal at R is expressed as

$$y_{\mathrm{R}} = \mathbf{h}_{\mathrm{SR}}(\mathbf{x}_i + \boldsymbol{\eta}_i^{\mathrm{t}}) + \eta_{\mathrm{R}}^{\mathrm{r}} + h_{\mathrm{RR}}(x_{\mathrm{R}} + \eta_{\mathrm{R}}^{\mathrm{t}}) + z_{\mathrm{R}}, \quad (1)$$

where $\mathbf{x}_i = [x_1 \ x_2 \ ... \ x_M]^T$ and $x_{\mathrm{R}}$ are, respectively, the intended transmitted signal vector from $M$ transmission antennas of S and the intended transmitted signal from transmission antenna of R, $i = 1, 2, ..., M$; $\mathbf{h}_{\mathrm{SR}} = [h_{1\mathrm{R}} \ h_{2\mathrm{R}} \ ... \ h_{M\mathrm{R}}]$ is the channel vector from $M$ transmission antennas of S to the reception antenna of R; $h_{\mathrm{RR}}$ is the SI channel at R; $\boldsymbol{\eta}_i^{\mathrm{t}} = [\eta_1^{\mathrm{t}} \ \eta_2^{\mathrm{t}} \ ... \ \eta_M^{\mathrm{t}}]^T$ is HI noise vector at S induced by the transmitter impairments; $\eta_{\mathrm{R}}^{\mathrm{t}}$ and $\eta_{\mathrm{R}}^{\mathrm{r}}$ are the HI noises coming from the transmitter and the receiver of R, respectively; $z_{\mathrm{R}}$ is Gaussian noise at R with zero mean and variance of $\sigma_{\mathrm{R}}^2$, i.e., $z_{\mathrm{R}} \sim \mathcal{CN}(0, \sigma_{\mathrm{R}}^2)$.

It is noted that, in the case of perfect transceiver hardwares, $\boldsymbol{\eta}_i^{\mathrm{t}} = \eta_{\mathrm{R}}^{\mathrm{r}} = \eta_{\mathrm{R}}^{\mathrm{t}} = 0$. On the other hand, the imperfect transceiver hardwares are from many factors such as in-phase/quadrature (I/Q) imbalance, nonlinearities of high power amplifier (HPA), low noise amplifier (LNA) and mixers [18], [19]. Different compensation algorithms have been introduced and applied at both transmitter and receiver to alleviate the transceiver HI. Unfortunately, the residual impairments still appear in the system as the Gaussian noises [18], [20], i.e., $\boldsymbol{\eta}_i^{\mathrm{t}} \sim \mathcal{CN}(0, (k_{\mathrm{S}}^{\mathrm{t}})^2 \frac{P_{\mathrm{S}}}{M})$, $\eta_{\mathrm{R}}^{\mathrm{t}} \sim \mathcal{CN}(0, (k_{\mathrm{R}}^{\mathrm{t}})^2 P_{\mathrm{R}})$ for the transmitting S and R, respectively, and $\eta_{\mathrm{R}}^{\mathrm{r}} \sim \mathcal{CN}(0, \|\mathbf{h}_{\mathrm{SR}}\|^2 (k_{\mathrm{R}}^{\mathrm{r}})^2 \frac{P_{\mathrm{S}}}{M})$ for the receiver R. Herein, $k_{\mathrm{S}}^{\mathrm{t}}$, $k_{\mathrm{R}}^{\mathrm{t}}$, and $k_{\mathrm{R}}^{\mathrm{r}}$ are, respectively, the HI levels at the transmitting S, R, and the receiving R; $\frac{P_{\mathrm{S}}}{M}$ refers to the average transmission power per antenna of S, and $P_{\mathrm{R}}$ denotes the average transmission power of R.

As shown in (1), the SI power before applying various SIC techniques is $h_{\mathrm{RR}}(x_{\mathrm{R}} + \eta_{\mathrm{R}}^{\mathrm{t}})$. However, after applying all SIC techniques, the RSI at FD relay (denoted by $I_{\mathrm{R}}$) can be modeled as Gaussian distributed random variable with zero mean and variance of $\sigma_{\mathrm{RSI}}^2$, i.e., $I_{\mathrm{R}} \sim \mathcal{CN}(0, \sigma_{\mathrm{RSI}}^2)$, where $\sigma_{\mathrm{RSI}}^2 = l^2 P_{\mathrm{R}}$ with $l$ is the SIC capability of FD relay [15], [16]. Consequently, the received signals at R becomes

$$y_{\mathrm{R}} = \mathbf{h}_{\mathrm{SR}}(\mathbf{x}_i + \boldsymbol{\eta}_i^{\mathrm{t}}) + \eta_{\mathrm{R}}^{\mathrm{r}} + I_{\mathrm{R}} + z_{\mathrm{R}}. \quad (2)$$

Next, R decodes this received signal, recodes, and forwards it to D. Then, the received signal at D is

$$\mathbf{y}_{\mathrm{D}} = \mathbf{h}_{\mathrm{RD}}(x_{\mathrm{R}} + \eta_{\mathrm{R}}^{\mathrm{t}}) + \boldsymbol{\eta}_j^{\mathrm{r}} + \mathbf{z}_{\mathrm{D}}, \quad (3)$$

where $\mathbf{h}_{\mathrm{RD}} = [h_{\mathrm{R}1} \ h_{\mathrm{R}2} \ ... \ h_{\mathrm{R}N}]^T$ is channel vector from the transmitting antenna of R to $N$ receiving antennas of D; $\boldsymbol{\eta}_j^{\mathrm{r}} = [\eta_1^{\mathrm{r}} \ \eta_2^{\mathrm{r}} \ ... \ \eta_N^{\mathrm{r}}]^T$ $(j = 1, 2, ..., N)$ is HI noise vector caused by the receiver impairment at D, i.e., $\boldsymbol{\eta}_j^{\mathrm{r}} \sim \mathcal{CN}(0, \|\mathbf{h}_{\mathrm{RD}}\|^2 (k_{\mathrm{D}}^{\mathrm{r}})^2 P_{\mathrm{R}})$ for the receiver D, where $k_{\mathrm{D}}^{\mathrm{r}}$ denotes the HI level at the receiver D; $\mathbf{z}_{\mathrm{D}}$ is Gaussian noise vector at D with zero mean and variance of $\sigma_{\mathrm{D}}^2$, i.e., $\mathbf{z}_{\mathrm{D}} \sim \mathcal{CN}(0, \sigma_{\mathrm{D}}^2)$.

From the received signals in (2) and (3), the instantaneous SINDR at R and D (denoted by $\gamma_{\mathrm{R}}$ and $\gamma_{\mathrm{D}}$ are, respectively, given by

$$\gamma_{\mathrm{R}} = \frac{\|\mathbf{h}_{\mathrm{SR}}\|^2 \frac{P_{\mathrm{S}}}{M}}{\|\mathbf{h}_{\mathrm{SR}}\|^2 (k_{\mathrm{S}}^{\mathrm{t}})^2 \frac{P_{\mathrm{S}}}{M} + \|\mathbf{h}_{\mathrm{SR}}\|^2 (k_{\mathrm{R}}^{\mathrm{r}})^2 \frac{P_{\mathrm{S}}}{M} + \sigma_{\mathrm{RSI}}^2 + \sigma_{\mathrm{R}}^2}$$
$$= \frac{\|\mathbf{h}_{\mathrm{SR}}\|^2 \frac{P_{\mathrm{S}}}{M}}{\|\mathbf{h}_{\mathrm{SR}}\|^2 k_{\mathrm{SR}}^2 \frac{P_{\mathrm{S}}}{M} + \sigma_{\mathrm{RSI}}^2 + \sigma_{\mathrm{R}}^2}, \quad (4)$$

$$\gamma_{\rm D} = \frac{\|\mathbf{h}_{\rm RD}\|^2 P_{\rm R}}{\|\mathbf{h}_{\rm RD}\|^2 (k_{\rm R}^{\rm t})^2 P_{\rm R} + \|\mathbf{h}_{\rm RD}\|^2 (k_{\rm D}^{\rm r})^2 P_{\rm R} + \sigma_{\rm D}^2} = \frac{\|\mathbf{h}_{\rm RD}\|^2 P_{\rm R}}{\|\mathbf{h}_{\rm RD}\|^2 k_{\rm RD}^2 P_{\rm R} + \sigma_{\rm D}^2}, \quad (5)$$

where $k_{\rm SR}^2 = (k_{\rm S}^{\rm t})^2 + (k_{\rm R}^{\rm r})^2$ is the aggregated HI level of both HI at the transmitting S ($k_{\rm S}^{\rm t}$) and the receiving R ($k_{\rm R}^{\rm r}$); $k_{\rm RD}^2 = (k_{\rm R}^{\rm t})^2 + (k_{\rm D}^{\rm r})^2$ is the aggregated HI level at both transmitting R ($k_{\rm R}^{\rm t}$) and receiving D ($k_{\rm D}^{\rm r}$).

Since the DF protocol is used at the relay, the end-to-end SINDR (denoted by $\gamma_{\rm e2e}$) of the FD relay system is presented as

$$\gamma_{\rm e2e} = \min\{\gamma_{\rm R}, \gamma_{\rm D}\}, \quad (6)$$

where $\gamma_{\rm R}$ and $\gamma_{\rm D}$ are given in (4) and (5), respectively.

## III. Performance Analysis

In this section, we mathematically derive the closed-form expression of the EC of the HI-FD relay system under the effects of both HI and RSI. Generally, the EC is calculated as

$$\mathcal{C} = \mathbb{E}\Big\{ \log_2(1+\gamma_{\rm e2e}) \Big\} = \int_0^\infty \log_2(1+\gamma_{\rm e2e}) f_{\gamma_{\rm e2e}}(\gamma) d\gamma, \quad (7)$$

where $\gamma_{\rm e2e}$ is the end-to-end SINDR of the FD relay system given in (6); $f_{\gamma_{\rm e2e}}(\gamma)$ is the probability density function (PDF) of $\gamma_{\rm e2e}$.

From (7), the EC of the HI-FD relay system is derived in the following Theorem 1.

**Theorem 1.** *Under the joint impacts of HI and RSI, the EC of HI-FD relay system using DF protocol and MRT/MRC techniques is expressed as*

$$\mathcal{C} = \frac{\pi}{2W\mu^2 \ln 2} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \sum_{w=1}^{W} \frac{\Psi^i \Phi^j}{i! j!} \times \frac{\alpha^{i+j}\sqrt{1-\beta^2}}{(1+\alpha)(1-k_{\rm SR}^2\alpha)^i (1-k_{\rm RD}^2\alpha)^j} \times \exp\Big( -\frac{\Psi\alpha}{1-k_{\rm SR}^2\alpha} - \frac{\Phi\alpha}{1-k_{\rm RD}^2\alpha} \Big), \quad (8)$$

*where* $\Psi = \frac{M(\sigma_{\rm RSI}^2+\sigma_{\rm R}^2)}{\Omega_1 P_{\rm S}}, \Phi = \frac{\sigma_{\rm D}^2}{\Omega_2 P_{\rm R}}, \mu = \max\{k_{\rm SR}, k_{\rm RD}\}$ $\alpha = \frac{1}{2\mu^2}(1+\beta)$, $\beta = \cos\Big(\frac{(2w-1)\pi}{2W}\Big)$, *$W$ is a complexity-accuracy trade-off parameters [22].*

*Proof:* Based on (7), after some mathematical manipulations, the EC is computed as

$$\mathcal{C} = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_{\gamma_{\rm e2e}}(x)}{1+x} dx, \quad (9)$$

where $F_{\gamma_{\rm e2e}}(x)$ is the cumulative distribution function (CDF) of $\gamma_{\rm e2e}$.

To derive the expression of EC, we first obtain $F_{\gamma_{\rm e2e}}(x)$. From the definition of $F_{\gamma_{\rm e2e}}(x)$, we have

$$F_{\gamma_{\rm e2e}}(x) = \Pr\{\gamma_{\rm e2e} < x\}. \quad (10)$$

Replacing $\gamma_{\rm e2e}$ from (6) into (10), $F_{\gamma_{\rm e2e}}(x)$ is now expressed as

$$F_{\gamma_{\rm e2e}}(x) = \Pr\{\min\{\gamma_{\rm R}, \gamma_{\rm D}\} < x\} = \Pr\{\gamma_{\rm R} < x\} + \Pr\{\gamma_{\rm D} < x\} - \Pr\{\gamma_{\rm R} < x\}\Pr\{\gamma_{\rm D} < x\}. \quad (11)$$

The first probability, i.e., $\Pr\{\gamma_{\rm R} < x\}$ in (11) can be presented as

$$\Pr\{\gamma_{\rm R} < x\} = \Pr\left\{ \frac{\|\mathbf{h}_{\rm SR}\|^2 \frac{P_{\rm S}}{M}}{\|\mathbf{h}_{\rm SR}\|^2 k_{\rm SR}^2 \frac{P_{\rm S}}{M} + \sigma_{\rm RSI}^2 + \sigma_{\rm R}^2} < x \right\}, \quad (12)$$

then becomes

$$\Pr\{\gamma_{\rm R} < x\} = \Pr\left\{ \|\mathbf{h}_{\rm SR}\|^2 \tfrac{P_{\rm S}}{M} (1 - k_{\rm SR}^2 x) < x(\sigma_{\rm RSI}^2 + \sigma_{\rm R}^2) \right\}. \quad (13)$$

To obtain the final expression from (13), we must consider two cases, i.e., $1 - k_{\rm SR}^2 x \le 0$ and $1 - k_{\rm SR}^2 x > 0$.

First, when $1 - k_{\rm SR}^2 x \le 0$, or $x \ge \frac{1}{k_{\rm SR}^2}$, the probability in (13) always holds, it is because $\|\mathbf{h}_{\rm SR}\|^2 \frac{P_{\rm S}}{M}(1-k_{\rm SR}^2 x) \le 0$ while $x(\sigma_{\rm RSI}^2 + \sigma_{\rm R}^2) > 0$. As a result, $\Pr\{\gamma_{\rm R} < x\} = 1$ when $x \ge \frac{1}{k_{\rm SR}^2}$.

Second, when $1 - k_{\rm SR}^2 x > 0$, or $x < \frac{1}{k_{\rm SR}^2}$, the probability in (13) is equivalent to

$$\Pr\{\gamma_{\rm R} < x\} = \Pr\left\{ \|\mathbf{h}_{\rm SR}\|^2 < \frac{Mx(\sigma_{\rm RSI}^2+\sigma_{\rm R}^2)}{P_{\rm S}(1-k_{\rm SR}^2 x)} \right\}. \quad (14)$$

Since $\|\mathbf{h}_{\rm SR}\|^2 = |h_{1\rm R}|^2 + |h_{2\rm R}|^2 + ... + |h_{M\rm R}|^2$, the CDF, $F(.)$ of $\|\mathbf{h}_{\rm SR}\|^2$ is expressed as

$$F_{\|\mathbf{h}_{\rm SR}\|^2}(x) = 1 - \exp\Big(-\frac{x}{\Omega_1}\Big) \sum_{i=0}^{M-1} \frac{1}{i!}\Big(\frac{x}{\Omega_1}\Big)^i, \quad x \ge 0, \quad (15)$$

where $\Omega_1 = \mathbb{E}\{|h_{1\rm R}|^2\} = \mathbb{E}\{|h_{2\rm R}|^2\} = ... = \mathbb{E}\{||h_{M\rm R}|^2\}$ is the average channel gain of $\rm S - R$ links.

Utilizing (15), (14) becomes

$$\Pr\{\gamma_{\rm R} < x\} = F_{\|\mathbf{h}_{\rm SR}\|^2}\left( \frac{Mx(\sigma_{\rm RSI}^2+\sigma_{\rm R}^2)}{P_{\rm S}(1-k_{\rm SR}^2 x)} \right) \quad (16)$$
$$= 1 - \exp\Big(-\frac{\Psi x}{1-k_{\rm SR}^2 x}\Big) \sum_{i=0}^{M-1} \frac{1}{i!}\left(\frac{\Psi x}{1-k_{\rm SR}^2 x}\right)^i.$$

After combining the two above cases, we have

$$\Pr\{\gamma_{\rm R} < x\} = \begin{cases} 1 - \exp\Big(-\frac{\Psi x}{1-k_{\rm SR}^2 x}\Big) \sum_{i=0}^{M-1} \frac{1}{i!}\Big(\frac{\Psi x}{1-k_{\rm SR}^2 x}\Big)^i, & x < \frac{1}{k_{\rm SR}^2}, \\ 1, & x \ge \frac{1}{k_{\rm SR}^2}. \end{cases} \quad (17)$$

Similarly, the probability $\Pr\{\gamma_{\rm D} < x\}$ is presented as

$$\Pr\{\gamma_{\rm D} < x\} = \begin{cases} 1 - \exp\Big(-\frac{\Phi x}{1-k_{\rm RD}^2 x}\Big) \sum_{j=0}^{N-1} \frac{1}{j!}\Big(\frac{\Phi x}{1-k_{\rm RD}^2 x}\Big)^j, & x < \frac{1}{k_{\rm RD}^2}, \\ 1, & x \ge \frac{1}{k_{\rm RD}^2}, \end{cases} \quad (18)$$

where $\Omega_2 = \mathbb{E}\{|h_{\rm R1}|^2\} = \mathbb{E}\{|h_{\rm R2}|^2\} = ... = \mathbb{E}\{||h_{\rm RN}|^2\}$ is the average channel gain of $\rm R - D$ links.

Now, $F_{\gamma_{\rm e2e}}(x)$ is expressed as

$$F_{\gamma_{\rm e2e}}(x) = \begin{cases} 1 - \exp\Big(-\frac{\Psi x}{1-k_{\rm SR}^2 x} - \frac{\Phi x}{1-k_{\rm RD}^2 x}\Big) \sum_{i=0}^{M-1} \\ \times \sum_{j=0}^{N-1} \frac{1}{i!j!}\Big(\frac{\Psi x}{1-k_{\rm SR}^2 x}\Big)^i \Big(\frac{\Phi x}{1-k_{\rm RD}^2 x}\Big)^j, x < \frac{1}{\mu^2}, \\ 1, x \ge \frac{1}{\mu^2}, \end{cases} \quad (19)$$

where $\mu = \max\{k_{\rm SR}, k_{\rm RD}\}$.

Then, substituting $F_{\gamma_{\rm e2e}}(x)$ from (19) into (9), we obtain (20). Applying [22, Eq. (25.4.30)], the integral in (20) is

$$\mathcal{C} = \frac{1}{\ln 2}\int_0^{\frac{1}{\mu^2}} \frac{1}{1+x}\exp\left(-\frac{\Psi x}{1-k_{\mathrm{SR}}^2 x}-\frac{\Phi x}{1-k_{\mathrm{RD}}^2 x}\right)\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\frac{1}{i!j!}\left(\frac{\Psi x}{1-k_{\mathrm{SR}}^2 x}\right)^i\left(\frac{\Phi x}{1-k_{\mathrm{RD}}^2 x}\right)^j dx$$

$$= \frac{1}{\ln 2}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\frac{\Psi^i\Phi^j}{i!j!}\int_0^{\frac{1}{\mu^2}}\frac{x^{i+j}}{(1+x)(1-k_{\mathrm{SR}}^2 x)^i(1-k_{\mathrm{RD}}^2 x)^j}\exp\left(-\frac{\Psi x}{1-k_{\mathrm{SR}}^2 x}-\frac{\Phi x}{1-k_{\mathrm{RD}}^2 x}\right)dx. \quad (20)$$

$$\int_0^{\frac{1}{\mu^2}}\frac{x^{i+j}}{(1+x)(1-k_{\mathrm{SR}}^2 x)^i(1-k_{\mathrm{RD}}^2 x)^j}\exp\left(-\frac{\Psi x}{1-k_{\mathrm{SR}}^2 x}-\frac{\Phi x}{1-k_{\mathrm{RD}}^2 x}\right)dx$$

$$= \frac{\pi}{2W\mu^2}\sum_{w=1}^{W}\frac{\alpha^{i+j}\sqrt{1-\beta^2}}{(1+\alpha)(1-k_{\mathrm{SR}}^2\alpha)^i(1-k_{\mathrm{RD}}^2\alpha)^j}\exp\left(-\frac{\Psi\alpha}{1-k_{\mathrm{SR}}^2\alpha}-\frac{\Phi\alpha}{1-k_{\mathrm{RD}}^2\alpha}\right). \quad (21)$$

calculated as in (21). Plugging (21) into (20), we obtain the EC of the HI-FD relay system as in (8). This completes the proof.

## IV. Numerical Results and Discussions

In this section, the mathematical expression of EC in previous section is used to evaluate the performance of HI-FD relay system. The Monte-Carlo simulations are also conducted to validate the theoretical analysis. In addition, the EC of HI-FD relay system is compared with that of ID-FD, HI-HD and ID-HD relay systems. In all evaluating scenarios, parameters are chosen as $\Omega_1 = \Omega_2 = 1$, $M = N = 4$, $k_{\mathrm{S}}^{\mathrm{t}} = k_{\mathrm{R}}^{\mathrm{t}} = k_{\mathrm{R}}^{\mathrm{r}} = k_{\mathrm{D}}^{\mathrm{r}} = k$, $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{D}}^2 = \sigma^2$, $P_{\mathrm{S}} = P_{\mathrm{R}} = P$. The average SNR is computed as the ratio between the average transmission power $P$ and the Gaussian noise power $\sigma^2$, i.e., $\mathrm{SNR} = P/\sigma^2$.

Fig. 2 illustrates the EC of the HI-FD relay system versus the average SNR in comparison with that of ID-FD, HI-HD, and ID-HD relay systems. The HI and RSI levels are $k = 0.1$ and $l = 0.15$, respectively. In Fig. 2, the analytical curve is obtained by using (8), while the marker denotes the Monte-Carlo simulation result. We can see that the EC of the HI-FD relay system is always higher than that of the HI-HD relay system for

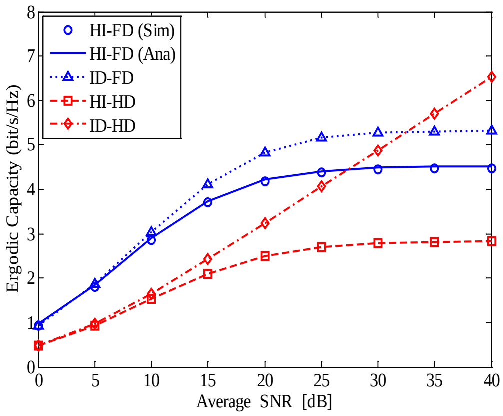


Figure 2. The EC of the considered HI-FD relay system versus the average SNR compared with those of ID-FD, HI-HD, and ID-HD relay systems, $k = 0.1, l = 0.15$.

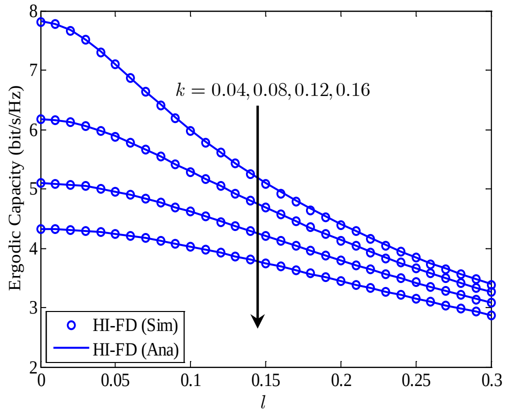


Figure 3. The impact of RSI on the EC of the considered HI-FD relay system for different HI levels, $\mathrm{SNR} = 30$ dB.

the investigated RSI level. Although both ECs of HI-FD and HI-HD relay systems reach the saturated, maximum capacities at $\mathrm{SNR} = 30$ dB, the peak EC of HI-FD relay system is 4.5 bit/s/Hz while it is 2.9 bit/s/Hz for HI-HD relay system. On the other hand, for the system with ideal hardwares, i.e., ID-FD and ID-HD relay systems, the EC of ID-FD relay system is not always higher than that of ID-HD relay system. Specifically, it is only higher when $\mathrm{SNR} < 35$ dB but is lower when $\mathrm{SNR} > 35$ dB. These results are reasonable because, in the high SNR regime, the impact of RSI becomes stronger; thus, the EC of ID-FD relay system is saturated while the EC of ID-HD relay system still increases. Furthermore, the HI significantly affects the ECs of both FD and HD relay systems. Particularly, at $\mathrm{SNR} = 30$ dB, ECs are 2.9, 4.5, 5, and 5.2 bit/s/Hz for HI-HD, HI-FD, ID-HD, and ID-FD relay systems, respectively.

Fig. 3 investigates the impact of RSI on the EC of the HI-FD relay system for different HI levels, i.e., $k = 0.04, 0.08, 0.12, 0.16$. As seen in Fig. 3, the effect of RSI on the EC of the HI-FD relay system is remarkable even with small $l$, especially when HI is insignificant. Specifically, the ECs reduce from 4.3, 5.1, 6.1, and 7.9 at $l = 0$ to 4.2, 4.9, 5.9, and 7 at $l = 0.05$ for $k = 0.16, 0.12, 0.08$ and 0.04, respectively. In the case of larger $k$, e.g., $k = 0.16$,

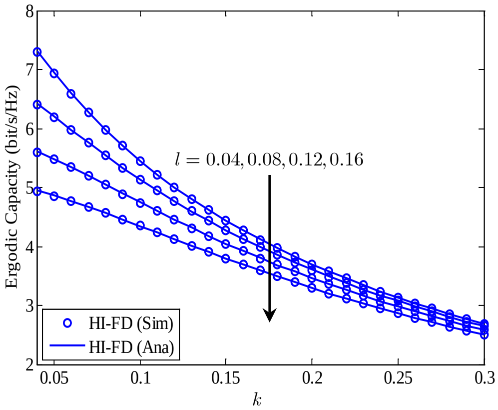


Figure 4. The EC of the considered HI-FD relay system versus the HI level for different RSI levels, $\text{SNR} = 30$ dB.

EC only reduces 0.1 bit/s/Hz (from 4.3 to 4.2) when $l$ increases from 0 to 0.05. However, when $k$ is very small, e.g., $k = 0.04$, EC reduces 0.9 bit/s/Hz (from 7.7 to 7) for that change of $l$. To sum up, the impact of RSI is stronger when HI is smaller. Therefore, more efforts to obtain a lower RSI level should be made to improve the performance of the HI-FD relay system.

Fig. 4 shows the EC of the HI-FD relay system versus the HI level $k$ for different RSI levels, i.e., $l = 0.04, 0.08, 0.12, 0.16$. Similar to Fig. 3, a strong impact of $k$ on the EC of HI-FD relay system can also be observed in Fig. 4. Also, the impact of $k$ is larger when $l$ is smaller. In particular, at $k = 0.05$, the ECs are 4.9, 5.5, 6.2 and 7 bit/s/Hz when $l = 0.16, 0.12, 0.08$ and 0.04. Therefore, the ECs in the cases $k = 0.05$ and $l = 0.16, 0.12, 0.08$ (refer to Fig. 4) are higher than those in the cases $l = 0.05$ and $k = 0.16, 0.12, 0.08$ (refer to Fig. 3). However, the EC in the case $k = 0.05$ and $l = 0.04$ is similar to that in the case $k = 0.04$ and $l = 0.05$. Therefore, besides developing other SIC techniques to reduce the RSI power, various solutions to mitigate HI should be exploited together to improve the performance of HI-FD relay system.

## V. Conclusion

The ergodic capacity is an important parameter that needs to be analyzed for wireless communication systems, especially for FD relay systems, thanks to the main benefit of the FD technique in enhancing the capacity. In this paper, we mathematically analyze the EC of an FD relay system with MRT/MC techniques under HI and RSI joint impact. From the obtained closed-form expression of the EC, the strong impacts of HI and RSI on the EC were evaluated to overcome the limitations of previous related works. Additionally, it was shown that the impact of HI is stronger when RSI is smaller and vice versa. The results in this paper are useful for the deployment and exploitation of practical FD relay systems.

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